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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 2, 2018/2019

**ETN3096 – DIGITAL SIGNAL PROCESSING**  
(CE, EE, LE, OPE, TE)

7 MARCH 2019  
2:30 PM – 4:30 PM  
(2 Hours)

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### INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 12 pages (including this cover page) with 4 Questions only and an appendix.
2. Attempt **ALL** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.

### Question 1

- (a) Consider a discrete-time system  $y[n] = 5n^3 x[n]$ .
- (i) Determine if the system is linear. [5 marks]
  - (ii) Determine if the system is time-invariant. [5 marks]
- (b) In the system shown in Figure Q1,  $h_1[n] = [\dots 0 \ 1 \ 2 \ 1 \ 0 \dots]$ .

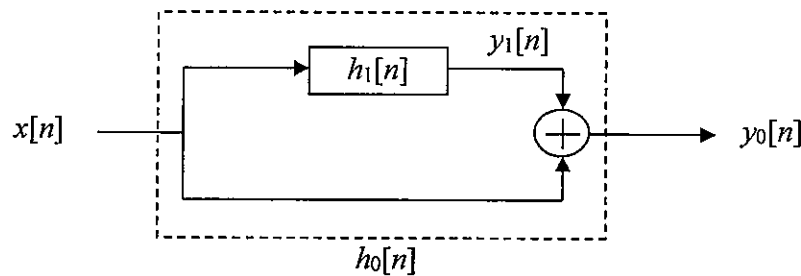


Figure Q1

- (i) For input  $x[n] = [\dots 0 \ 4 \ 3 \ 2 \ 0 \dots]$ , determine  $y_1[n]$  and  $y_0[n]$  for  $n = 0$  to 5. [10 marks]
- (ii) Determine the impulse response of the overall system  $h_0[n]$ . State if it is causal and justify your answer. [5 marks]

### Question 2

- (a) Given that  $H(z) = \frac{1}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)}$ ,  $\frac{1}{3} < |z| < \frac{1}{2}$ , do the following:
- (i) Find the inverse  $z$ -transform of  $H(z)$ . [6 marks]
  - (ii) If  $H(z)$  represents a transfer function, determine the corresponding linear constant coefficient difference equation. [4 marks]
- (b) (i) Given a sequence  $x[n] = [1 \ 0 \ 0 \ 2]$ , evaluate the 4-point Discrete Fourier Transform (DFT) of the sequence  $x[n]$ . Compute all four DFT samples. [6 marks]
- (ii) Let  $w[n]$  be the circularly shifted signal of  $x[n]$  where  $w[n] = x[n-2]_4$ . Sketch the signal  $w[n]$  and compute the 4-point DFT of  $w[n]$  by using the **properties of DFT**. All the 4 DFT samples must be computed. [9 marks]

Continued ...

**Question 3**

- (a) Design a 3-tap Finite Impulse Response (FIR) lowpass filter with a cutoff frequency of **1500 Hz** and a sampling rate of **8000 sample/s**.
- (i) Determine the filter coefficients of the filter using a *rectangular* window function. [9 marks]
  - (ii) Write down the transfer function and difference equation of the filter. [3 marks]
- (b) A lowpass FIR filter has the following specifications:
- Passband: 0 – 800 Hz
  - Stopband: 1200 – 4000 Hz
  - Stopband attenuation: 40 dB
  - Passband ripple: 0.1 dB
  - Sampling rate: 8000 Hz
- Determine the window method, the FIR filter length, and the cutoff frequency to be used in the design. [6 marks]

Continued ...

- (c) Figure Q3 shows a simple noise canceller with the use of one-tap adaptive filter. The adaptive filter equation is given in (1) and weight update equation for the least mean square (LMS) algorithm is given in (2).
- Describe briefly the algorithm used in adaptive filtering. [7 marks]
  - Explain the difference between adaptive FIR filter and non-adaptive FIR filter. [3 marks]
  - Complete Table Q3 for  $n = 0$  and 1. Calculation steps must be shown. [5 marks]

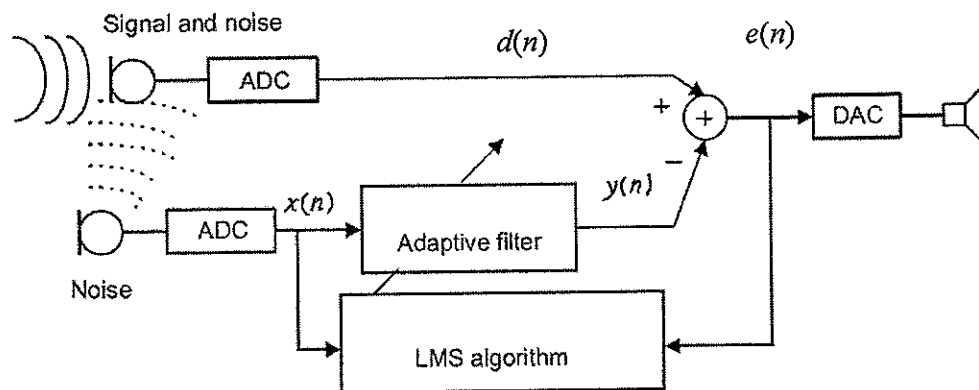


Figure Q3

$$y(n) = w(n)x(n) \quad (1)$$

$$w(n+1) = w(n) + 0.02 e(n) x(n) \quad (2)$$

Table Q3: Adaptive Filter Computation

Iteration $n$	Signal corrupted with noise $d(n)$	Noise signal $x(n)$	Filter output $y(n)$	Error signal $e(n)$	Filter weight $w(n)$
0	-1.310	-0.150			0.200
1	1.221	0.212			

Continued ...

**Question 4**

Sketch the structures of an Infinite Impulse Response (IIR) filter with the transfer function

$$H(z) = \frac{0.4(1 - z^{-1})}{(1 + 0.4z^{-1})(1 + 0.8z^{-1})}$$

in the following forms:

- |                     |           |
|---------------------|-----------|
| (i) Direct Form I   | {6 marks} |
| (ii) Direct Form II | [5 marks] |
| (iii) Cascade Form  | [6 marks] |

**Continued ...**

## APPENDIX

## Discrete-time Fourier transform

## Properties

Property	$x[n], y[n]$	$X(e^{jw}), Y(e^{jw})$
Linearity	$ax[n] + by[n]$	$aX(e^{jw}) + bY(e^{jw})$
Time shifting	$x[n - n_0]$	$e^{-jwn_0} X(e^{jw})$
Frequency shifting	$e^{jw_0 n} x[n]$	$X(e^{j(w-w_0)})$
Time reversal	$x[-n]$	$X(e^{-jw})$
Differentiation	$n^k x[n]$	$(j)^k \frac{d^k}{dw^k} X(e^{jw})$
Convolution	$x[n] * y[n]$	$X(e^{jw})Y(e^{jw})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} (X(e^{j\theta}) * Y(e^{jw}))$

## Common pairs

$x[n]$	$X(e^{jw})$
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-jwn_0}$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(w + 2\pi k)$
$e^{jw_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(w - w_0 + 2\pi k)$
$u[n]$	$\frac{1}{1 - e^{-jw}} + \sum_{k=-\infty}^{\infty} \pi\delta(w + 2\pi k)$
$a^n u[n],  a  < 1$	$\frac{1}{1 - ae^{-jw}}$
$(n + 1)a^n u[n],  a  < 1$	$\frac{1}{(1 - ae^{-jw})^2}$
$\frac{\sin w_c n}{\pi n}$	$X(e^{jw}) = \begin{cases} 1, &  w  < w_c \\ 0, & w_c <  w  \leq \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin w(M + 1)/2}{\sin w/2} e^{-jwM/2}$

## z-transform

### Properties

Properties	Sequence	z-transform	ROC
	$x[n]$	$X(z)$	$R_x$
	$x_1[n]$	$X_1(z)$	$R_{x1}$
	$x_2[n]$	$X_2(z)$	$R_{x2}$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	Contains $R_{x1} \cap R_{x2}$
Time shifting	$x[n-m]$	$z^{-m} X(z)$	$R_x$ except for the possible addition or deletion of the origin or infinity.
Multiplication by an exponential sequence	$a^n x[n]$	$X(z/a)$	$ a R_x$
Differentiation	$nx[n]$	$-z \frac{dX(z)}{dz}$	$R_x$ except for the possible addition or deletion of the origin or infinity.
Conjugate	$x^*[n]$	$X^*(z^*)$	$R_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/R_x$
Convolution	$x_1[n] * x_2[n]$	$X_1(z) X_2(z)$	Contains $R_{x1} \cap R_{x2}$

### Common pairs

Sequence	Transform	ROC
$\delta[n]$	1	All $z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
$\delta[n - m]$	$z^{-m}$	All $z$ except 0 (if $m > 0$ ) or $\infty$ (if $m < 0$ )
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $

## Finite Impulse Response (FIR) Filters

Ideal impulse responses for standard FIR filters

Filter Type	Ideal Impulse Response $h(n)$
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi}, & n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi}, & n \neq 0 \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi - \Omega_c}{\pi}, & n = 0 \\ \frac{-\sin(\Omega_c n)}{n\pi}, & n \neq 0 \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H - \Omega_L}{\pi}, & n = 0 \\ \frac{\sin(\Omega_H n) - \sin(\Omega_L n)}{n\pi}, & n \neq 0 \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi - (\Omega_H - \Omega_L)}{\pi}, & n = 0 \\ \frac{-\sin(\Omega_H n) + \sin(\Omega_L n)}{n\pi}, & n \neq 0 \end{cases}$

FIR filter length estimation using window functions

Window Type	Window Function $w(n), -M \leq n \leq M$
Rectangular	1
Hanning	$0.5 + 0.5 \cos\left(\frac{n\pi}{M}\right)$
Hamming	$0.54 + 0.46 \cos\left(\frac{n\pi}{M}\right)$
Blackman	$0.42 + 0.5 \cos\left(\frac{n\pi}{M}\right) + 0.08 \cos\left(\frac{2n\pi}{M}\right)$

Normalized Transition Width $\Delta f = \frac{ f_{stop} - f_{pass} }{f_{sampling}}$			
Type of Window	Window Length $N$	Stopband Attenuation (dB)	Passband Ripple (dB)
Rectangular	$N = 0.9/\Delta f$	21	0.7416
Hanning	$N = 3.1/\Delta f$	44	0.0546
Hamming	$N = 3.3/\Delta f$	53	0.0194
Blackman	$N = 5.5/\Delta f$	74	0.0017



## Bilinear Transformation (BLT)

### 1. Frequency prewarping

Let  $\omega_a$  denote the analog frequency marked on the  $j\omega$ -axis on the  $s$ -plane, and  $\omega_d$  denote the digital frequency marked on the unit circle in the  $z$ -plane.

For the lowpass filter and highpass filter:

$$\omega_a = \frac{2}{T} \tan\left(\frac{\omega_d T}{2}\right)$$

For the bandpass filter and bandstop filter:

$$\omega_{al} = \frac{2}{T} \tan\left(\frac{\omega_l T}{2}\right), \quad \omega_{ah} = \frac{2}{T} \tan\left(\frac{\omega_h T}{2}\right)$$

and  $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}$ ,  $W = \omega_{ah} - \omega_{al}$ .

### 2. Prototype transformation using the lowpass prototype $H_p(s)$

From lowpass to lowpass:  $H(s) = H_p(s) \Big|_{s=\frac{s}{\omega_a}}$

From lowpass to highpass:  $H(s) = H_p(s) \Big|_{s=\frac{\omega_a}{s}}$

From lowpass to bandpass:  $H(s) = H_p(s) \Big|_{s=\frac{s^2 + \omega_0^2}{sW}}$

From lowpass to bandstop:  $H(s) = H_p(s) \Big|_{s=\frac{sW}{s^2 + \omega_0^2}}$

where  $\omega_a$  denotes the analog frequency,  $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}$ ,  $W = \omega_{ah} - \omega_{al}$ .

### 3. Substitute the BLT to obtain the digital filter

$$H(z) = H(s) \Big|_{s=\frac{2}{T} \frac{z-1}{z+1}}$$

## Conversion from Analog Filter Specifications to Lowpass Prototype Specifications

### Analog Filter Specifications    Lowpass Prototype Specifications

Lowpass:  $\omega_{ap}, \omega_{as}$

$$v_s = \frac{\omega_{as}}{\omega_{ap}}$$

Highpass:  $\omega_{ap}, \omega_{as}$

$$v_s = \frac{\omega_{ap}}{\omega_{as}}$$

Bandpass:  $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$

$$\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$$

$$v_s = \frac{\omega_{ash} - \omega_{asl}}{\omega_{aph} - \omega_{apl}}$$

Bandstop:  $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$

$$\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$$

$$v_s = \frac{\omega_{aph} - \omega_{apl}}{\omega_{ash} - \omega_{asl}}$$

$\omega_{ap}$ , passband frequency edge;  $\omega_{as}$ , stopband frequency edge

$\omega_{apl}$ , lower cutoff frequency in passband;  $\omega_{aph}$ , upper cutoff frequency in passband

$\omega_{asl}$ , lower cutoff frequency in stopband;  $\omega_{ash}$ , upper cutoff frequency in stopband

$\omega_0$ , geometric center frequency

### Closed-Form Expression for Some Useful Series

$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$ $\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$ $\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$ $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad  a  < 1$	$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2}$ $\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$ $\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$
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### Digital Butterworth and Chebyshev Filter Designs

With the given passband ripple  $A_p$  dB at the normalized passband frequency edge  $v_p = 1$ , and the stopband attenuation  $A_s$  dB at the normalized stopband frequency edge  $v_s$ ,  $\varepsilon$  is the absolute ripple specification

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

Butterworth lowpass prototype order

$$n \geq \frac{\log_{10} \left( \frac{10^{0.1A_s} - 1}{\varepsilon^2} \right)}{|2 \log_{10}(v_s)|}$$

Chebyshev lowpass prototype order

$$n \geq \frac{\cosh^{-1} \left( \sqrt{\frac{10^{0.1A_s} - 1}{\varepsilon^2}} \right)}{\cosh^{-1}(v_s)}$$

where  $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$

3-dB Butterworth Lowpass Prototype Transfer Functions ( $\varepsilon = 1$ )

$n$	$H_p(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2 + 1.4142s + 1}$
3	$\frac{1}{s^3 + 2s^2 + 2s + 1}$

Chebyshev Lowpass Prototype Transfer Functions with 1dB Ripple ( $\varepsilon = 0.5088$ )

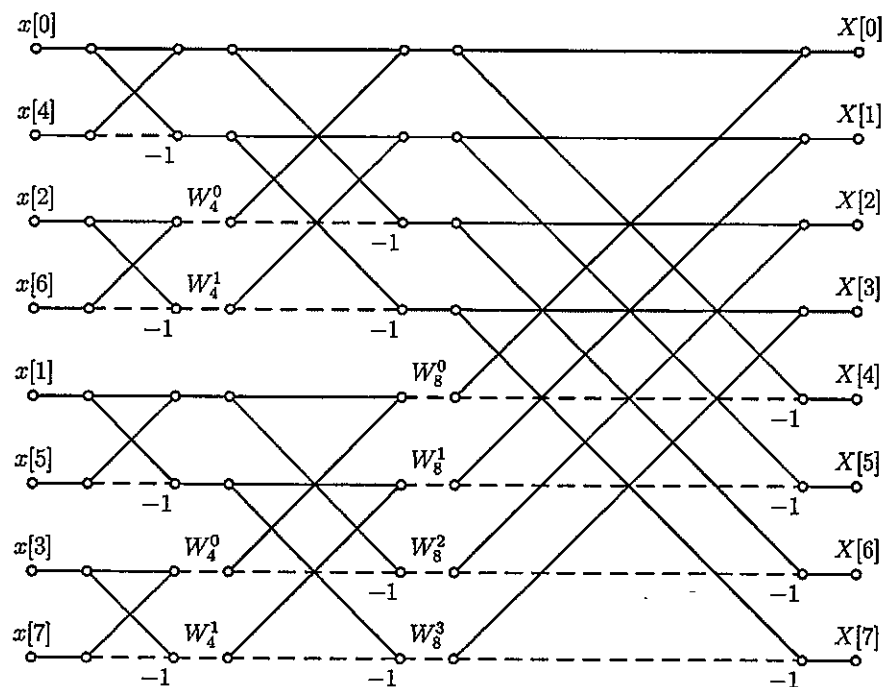
$n$	$H_p(s)$
1	$\frac{1.9652}{s+1.9652}$
2	$\frac{0.9826}{s^2 + 1.0977s + 1.1025}$
3	$\frac{0.4913}{s^3 + 0.9883s^2 + 1.2384s + 0.4913}$

## Discrete Fourier Transform

### Properties

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0 \rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0 \rangle_N]$
Time reversal	$x[\langle -n \rangle_N]$	$X[\langle -k \rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k \rangle_N]$
Convolution	$x[n] \otimes y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \otimes Y[k]$

### The decimation-in-time fast Fourier transform



End of Paper

